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Forecasting currency risk of enterprise's asset portfolio using the Monte Carlo simulation

Introduction

Currency risk can have serious consequences for an enterprise. Even in short time intervals, exchange rate changes can be significant. Enterprises operating simultaneously in several foreign currencies have to be aware of the interdependencies between exchange rates changes. They have to treat their positions denominated in foreign currencies as an asset portfolio (as far as it is possible in terms of payments synchronization). Calculating related risk separately for every position in foreign currency (e.g. for several foreign receivables) might lead to the overestimation of risk in a given forecasting horizon. The fact that the probability distribution that describes the changes in a particular currency at best may differ from the others among the considered set of currencies, as well as it may not be necessarily a normal distribution, seems to be another important issue.

The primary aim of the paper is to indicate that an enterprise having a portfolio that contains several positions denominated in different foreign currencies should perform risk measurement in a desired time horizon, assuming both that the exchange rates changes are interdependent and that the exchange rates changes are not necessarily normally distributed. Recapitulating the above, risk forecasting needs a proper measurement method. The Monte Carlo method enables its users to reflect currencies' changes using different probability distributions for different currencies at the same time as well as assuming the interdependencies between them. Using the Monte Carlo method, one is able to build a model which can be very clear and descriptive at the same time. The secondary aim of the paper is then to propose a concept of such a model.

1. The risk of the portfolio of currency positions

An enterprise having currency positions denominated in several different foreign currencies should treat them as a portfolio of assets. The portfolio volatility results in its value at risk (VaR). VaR is a loss in a given time horizon that will not be exceeded with a given confidence level (Jorion, 2007). In terms of securing a portfolio of currency positions by risk retention, VaR is an amount that can cover the adverse changes of this portfolio with the desired confidence level. The VaR of a portfolio of currency positions is not simply the sum of its elements and their VaRs in a given time horizon. The reason for this is the interdependencies between exchange rates changes of the currencies. A proper risk measurement method is then essential.

The method of historical simulation admittedly enables the quantification of risk related to a currency position or a portfolio of several currency positions. Unfortunately, the forecasting horizon is determined by the frequency of the historical time series in use. Therefore, daily, weekly or monthly time series enable forecasting in a horizon of one day, one week or one month ahead respectively. Using a historical simulation may result in problems when the time series is simply too short[1].

Forecasting in any given time horizon with historical time series taken into account is possible using a geometric Brownian motion (GBM). The common approach to GBM assumes that logarithmic changes of a financial category (e.g. stock price, exchange rate, commodity price) are normally distributed and result in a future value that has a log-normal probability distribution. Calculating the average value and the standard deviation of historical logarithmic changes of a financial category enables the calculation of its future expected value and future standard deviation in any given time horizon. Calculating VaR is also possible by using the inverse function for the cumulative distribution function of the log-normal probability distribution. The inverse function then gives the quantile of a future value probability distribution. (Brigo, Dalessandro, Neugebauer and Triki, 2007; Vose, 2008; Glasserman, 2004).

The probability distribution of a financial category future value can be obtained using the Monte Carlo simulation. The simulation is then assumed to be a tool of a complex scenario analysis (Kroese, Brereton, Taimre, Botev, 2014). The essence of the simulation is to build a financial model of a time series enabling the calculation of the future value of a financial category (1).

$$P_{t+1} = P_t exp[G_{BF}(Par_1, Par_2, \dots, Par_m)] \tag{1}$$

[1] Of course, daily data can be easily filtered into weekly, monthly, quarterly data and so on. The longer the interval the data is filtered into, the less data points remain in the historical time series.

where:
G_{BF} () – the inverse function for the cumulative distribution function of a probability distribution that fits to historical logarithmic changes of a financial category at best,
Par_1, Par_2, …, Par_m – the parameters of the inverse function,
P_t – the value of a financial category in the current period,
P_{t+1} – the value of a financial category in the next period.

The logarithmic changes of a financial category overtime are generated randomly resulting in one of the possible scenarios of its future value (Vose, 2008). This procedure is repeated multiple times enabling one to determine the probability distribution of the future value in the considered time horizon (more on the essence of the Monte Carlo mechanism in: Vose, 2008; Rees, 2008; Gentry and Pyhrr, 1973; Hertz, 1964). The basic advantage of the Monte Carlo approach to GBM is the possibility to assume any theoretical probability distribution for reflecting the changes of a financial category over time.

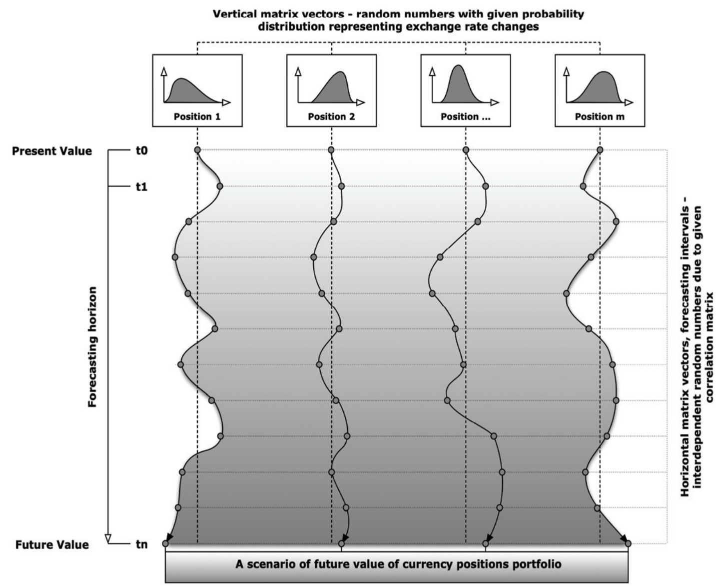
Goodness-of-fit statistics like Chi-Square, Kolmogorov-Smirnov and Anderson-Darling or information criteria like Schwarz, Akaike and Hannan-Quin are among the useful tools that enable fitting a theoretical probability distribution to historical data (to the empirical probability distribution) (for more see Vose, 2008).

Building a financial model that depicts the behaviour of a portfolio that consists of currency positions in a desired time horizon is essential in the Monte Carlo approach (Figure 1). The model has to be able to reflect the behaviour of a correlated time series. The model itself is a matrix of interdependent random numbers. Every vertical vector of the matrix consists of random numbers with a given probability distribution (it can be a normal distribution or another probability distribution that fits to the historical logarithmic changes of a particular exchange rate at best). The number of vector elements depends on the chosen time horizon (n) and the forecasting interval (t). The subsequent horizontal vectors of the matrix reflect the subsequent forecasted intervals. The horizontal vectors are the scenarios of the future values of the portfolio and its elements. The number of the vertical vectors depends on the number of currency positions taken into account. Every time the model is recalculated, a new scenario of the portfolio and its future value is generated. Multiple model recalculation and scenarios generation enable one to obtain the probability distribution of the portfolio future value. The essential element of the model is a mechanism that enables recalculating the model and storing scenarios. In case the model is a spreadsheet, the mechanism can be automatized using object-oriented programming (e.g. Visual Basic for Applications in Microsoft Excel).

The vectors of interdependent random numbers with different given probability distribution types that reflect a given correlation matrix can be obtained in a spreadsheet by Cholesky's decomposition (Kaczmarzyk, 2016). The implementation of a user defined function that results with a matrix M which multiplied by the transposed matrix M^T gives the correlation matrix Σ (Wilmott, 2006) is then essential. The correlation matrix in this approach has to be a positive-definite one (Korn, Korn, Kroisandt, 2010) which can be a significant restraint in some situations. The generation of correlated time series can be automatized in a spreadsheet by using specialized add-ins (e.g. Palisade @Risk). Such a solution is particularly convenient, especially when the probability distributions that fit to the historical time series at best are to be taken into account in a risk forecast. Such add-ins automatize the fitting of probability distributions which is particularly demanding and time-consuming.

Figure 1.

A model that enables forecasting the future value of a portfolio of currency positions using the Monte Carlo simulation



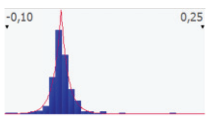
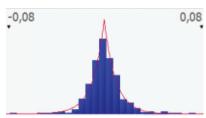
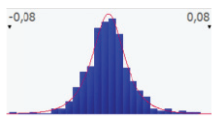
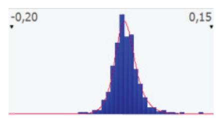
Source: own study.

2. Case study

An enterprise possesses receivables denominated in four foreign currencies (CHF, EUR, GBP, USD). Their total worth in domestic currency is 10,000 PLN according to the average exchange rate from 31 Dec 2016 (2,500 PLN for each receivable). It has been assumed that the enterprise is going to receive payments after one year (52 weeks). The enterprise thus has a portfolio of receivables and intends to secure it making a financial reserve. The reserve has to cover the possible adverse changes of the portfolio that could arise from adverse exchange rates changes with a 99% level of confidence. In order to get the probability distribution of the portfolio's future value, an assumption of GBM for exchange rates behavior was made. The Monte Carlo approach to GBM was chosen as the method of calculation. The simulation was performed using both the assumption of normal probability distribution as well as the assumption of best fitting probability distribution. In order to achieve precise outcomes, 100 000 iterations were made for each simulation[2]. The distributions were fitted using the Akaike information criteria (AIC). The parameters of the distributions were obtained on the basis of 521 historical logarithmic changes of foreign currencies from 5 Jan 2007 to 30 Dec 2016 (Table 1).

Table 1.

The result of probability distribution fitting

CHFPLN	**EURPLN**	**GBPPLN**	**USDPLN**
The best fitting probability distribution to exchange rate logarithmic changes according to AIC:			
Laplace	**Laplace**	**HypSecant**	Loglogistic
			
RiskLaplace (–0,00085897;0,017752)	RiskLaplace (–0,00056193;0,013083)	RiskHypSecant (–0,00065269;0,017404)	RiskLoglogistic (–0,1897;0,18908;16,706)
The fitting was performed using Palisade @RISK 7.5.2			

Source: own study.

First, perfect positive correlations were assumed between exchange rates changes – rank correlation coefficients of +1.0. Such interdependencies are not real anyway (Table 2).

In the case of perfect positive correlations between exchange rates changes, the expected value, standard deviation and absolute VaR are equal to or very close to the sum of the expected values, the sum of standard deviations or the sum of VaRs of the portfolio elements. The expected value of the portfolio is 10,316.04 PLN for the normal probability distributions and 9,875.10 PLN for the best fitting probability distributions. The dispersion of the potential future values of the portfolio measured with standard deviation is 1,356.67 PLN and 1,238.30 PLN respectively. The 1% quantile of the portfolio value is equal to 7,559.50 PLN for the normal probability distributions reflecting the changes of exchange rates.

[2] The simulations were performed using Palisade @RISK 7.5.2 in Microsoft Excel 2016.

When it comes to the best fitting probability distributions, the 1% quantile of the portfolio future value is a little bit lower – 7,345.06 PLN. This means that the portfolio future value is going to be higher than 7,559.50 PLN and 7,345.06 PLN respectively with a 99% confidence. The absolute VaR is 2,440.50 PLN when it comes to normally distributed currency changes and 2,654.94 PLN when the best fitting probability distributions are in use. The assumption of normal probability distribution leads to the absolute VaR 8.08% lower in comparison to the best fitting distribution assumption. It has to be emphasized that if an enterprise made financial reserves for every currency position separately, it would behave exactly the same by making a reserve for a portfolio and assuming unreal perfect positive correlations between its elements.

Table 2.

Portfolio absolute Value at Risk (rank correlation coefficients +1,0)

Measure	**Probability distribution**	**CHF**	**EUR**	**GBP**	**USD**	**Portfolio**	**The sum for portfolio elements**
Expected value	N*	2,663.57	2,543.52	2,491.78	2,617.16	10,316.04	10,316.04
	BF*	2,410.46	2,438.30	2,435.19	2,591.15	9,875.10	9,875.10
Standard deviation	N	385.40	247.53	312.63	411.26	1,356.67	1,356.82
	BF	309.91	232.50	306.92	391.78	1,238.30	1,241.11
1% Quantile	N	1,888.02	2,021.23	1,850.37	1,799.88	7,559.50	7,559.50
	BF	1,771.38	1,941.31	1,800.32	1,822.48	7,345.06	7,335.50
*VaR_A 1%	N	611.98	478.77	649.63	700.12	2,440.50	2,440.50
	BF	728.62	558.69	699.68	677.52	2,654.94	2,664.50

* N – normal probability distribution,
BF – best fitting probability distribution AIC,
VaR_A – absolute value at risk.

Source: own study.

If the enterprise made a forecast assuming independent exchange rate changes, it would admittedly obtain the same expected value of the portfolio. The enterprise would identify significantly lower volatility at the same time[3]. The absolute value at risk would account for 1,176.86 PLN assuming normal probability distributions and 1,493.27 PLN in the case of the best fitting ones. The normal probability distribution assumption would lead then to an absolute VaR 21.19% lower (Table 3). Regardless of the final distribution assumption, the absolute VaR would be significantly lower because of the independent exchange rate changes.

To recap the above, if an enterprise secures a portfolio taking into account VaR calculated separately for every currency position or a portfolio VaR calculated

[3] In accordance with the principles of portfolio theory (K. Jajuga and T. Jajuga, 2012; Elton, Gruber, Brown and Goetzmann, 2003).

assuming perfect positive correlations, the enterprise will overestimate risk and make a financial reserve higher than necessary. The zero correlation assumption will lead to an underestimated financial reserve, of course when the real correlations are positive.

Table 3.
Portfolio absolute Value at Risk (rank correlation coefficients 0,0)

Measure	Probability distribution	CHF	EUR	GBP	USD	Portfolio	The sum for portfolio elements
Expected value	N*	2,663.50	2,543.40	2,491.52	2,616.85	10,315.27	10,315.27
	BF*	2,410.57	2,438.35	2,435.19	2,591.00	9,875.11	9,875.11
Standard deviation	N	384.98	246.33	310.53	409.25	689.25	1,351.08
	BF	310.85	233.07	306.94	390.68	632.35	1,241.53
1% Quantile	N	1,887.03	2,019.55	1,858.20	1,798.90	8,823.14	7,563.68
	BF	1,769.17	1,943.28	1,801.56	1,829.50	8,506.73	7,343.52
*VaR_A 1%	N	612.97	480.45	641.80	701.10	1,176.86	2,436.32
	BF	730.83	556.72	698.44	670.50	1,493.27	2,656.48

* N – normal probability distribution,
BF – best fitting probability distribution AIC,
VaR_A – absolute value at risk.

Source: own study.

The enterprise should necessarily derive the probability distribution of the portfolio future value assuming real rank correlation coefficients between currency positions. In the case being considered in the paper, the rank correlation coefficients obtained on the basis of historical data are positive. Their intensity has to be perceived as moderate (Table 4).

Table 4.
Historical rank correlation coefficients between exchange rate changes

ρ	CHF	EUR	GBP	USD
CHF	1			
EUR	0.830048163	1		
GBP	0.586837273	0.65309339	1	
USD	0.665669808	0.716726164	0.744413756	1

Source: own study.

Assuming historical correlation coefficients between exchange rates changes lead to the standard deviation of 1,188.28 PLN for the normal distributions and 1,085,88 PLN for the best fitting ones (Table 5). The 1% quantile of the future portfolio value is equal to 7,860.73 PLN when it comes to the normally distributed exchange rates changes and 7,642.08 PLN when it comes to the best fitting distributions. This means that in one year's time, the portfolio value is going to be higher than 7,860.73 PLN and 7,642.08 PLN respectively, with a 99% confidence level. As a consequence, the absolute VaR for the considered portfolio of currency positions is equal to 2,139.27 PLN when exchange rates changes are normally distributed and 2,357.92 PLN when they are reflected by the best fitting distributions. The assumption of normal distributions leads to absolute VaR 9.27% lower than in the case of the best fitting ones.

Table 5.

Portfolio absolute Value at Risk (historical rank correlation coefficients)

Measure	Probability distribution	CHF	EUR	GBP	USD	Portfolio	The sum for portfolio elements
Expected value	N*	2,663.39	2,543.41	2,491.68	2,616.90	10,315.39	10,315.39
	BF*	2,410.48	2,438.31	2,435.11	2,591.22	9,875.12	9,875.12
Standard deviation	N	384.07	246.34	311.92	409.59	1,188.28	1,351.91
	BF	310.19	232.80	306.25	392.68	1,085.88	1,241.93
1% Quantile	N	1,889.65	2,020.60	1,849.01	1,803.70	7,860.73	7,562.97
	BF	1,774.39	1,947.84	1,800.99	1,823.44	7,642.08	7,346.66
*VaR_A 1%	N	610.35	479.40	650.99	696.30	2,139.27	2,437.03
	BF	725.61	552.16	699.01	676.56	2,357.92	2,653.34

* N – normal probability distribution,
BF – best fitting probability distribution AIC,
VaR_A – absolute value at risk.

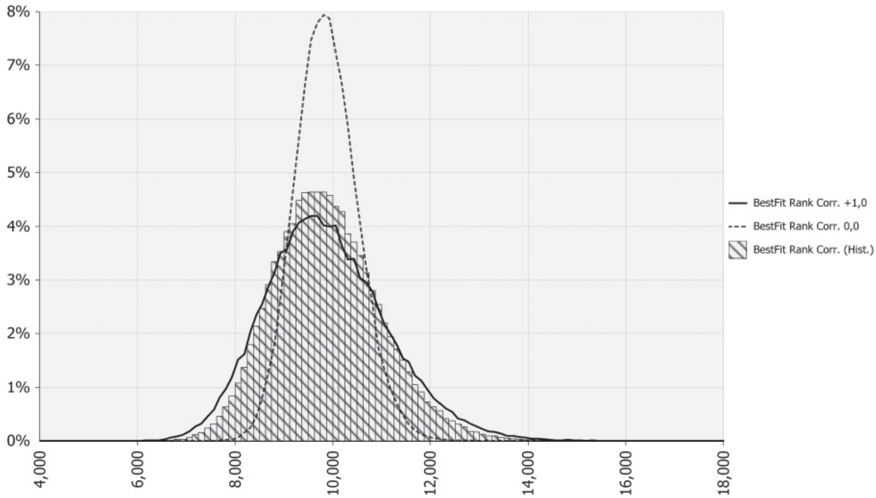
Source: own study.

It should be noted that an enterprise must be aware of its receivables or liabilities denominated in foreign currencies and their tendency to change interdependently. Regarding the related payment horizons and possible synchronization, these currency positions should be treated as a portfolio of assets or liabilities (Figure 2). On the other hand, the enterprise should be conscious that assuming the normal probability distributions for exchange rates changes brings different results when compared to the use of the best fitting ones (Figure 3).

It is also worth emphasizing that precise currency risk simulation is the basis for effective decision-making in terms of currency risk control. For some combinations of currencies, a portfolio of receivables or liabilities can have significantly lower risk than the respective currency positions treated separately. It may happen that risk retention can be an acceptable solution for controlling the risk. It should be also noted that the Monte Carlo simulation as an approach to the risk measurement of a portfolio of currency positions is at the same time an easy to implement and a very clear solution.

Figure 2.

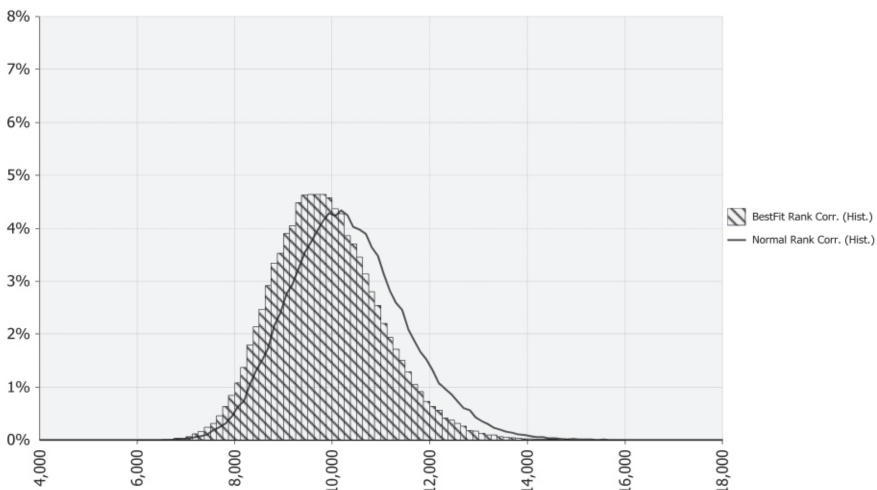
The probability distributions of future value regarding different correlation assumptions



Source: own study.

Figure 3.

The impact of the best fitting probability assumption on the future value of the portfolio



Source: own study.

Conclusions

Forecasting risk of a portfolio of currency positions is a complex problem. Professional risk measurement should bring information in terms of a portfolio VaR. Historical simulation seems to be a good solution. Unfortunately, it has forecasting limitations, especially if the available historical time series are too short. This usually happens when the chosen forecasting horizon is relatively long, e.g. half a year or a year, in which case an enterprise may use the Monte Carlo simulation. This approach enables an enterprise to assume interdependencies between exchange rates changes. It also allows the enterprise to assume that these changes are not necessarily normally distributed and could be better reflected with the best fitting (to historical changes) probability distributions. The Monte Carlo approach needs a computer financial model that can be developed even in a spreadsheet.

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Summary

The aim of the paper is to point out that the Monte Carlo simulation is an easy and flexible approach when it comes to forecasting risk of an asset portfolio. The case study presented in the paper illustrates the problem of forecasting risk arising from a portfolio of receivables denominated in different foreign currencies. Such a problem seems to be close to the real issue for enterprises offering products or services on several foreign markets. The changes in exchange rates are usually not normally distributed and, moreover, they are always interdependent. As shown in the paper, the Monte Carlo simulation allows for forecasting market risk under such circumstances.

Key words: currency risk, forecasting, enterprise, Monte Carlo method